Coupling of stochastic differential equations and real interpolation

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We consider a fully path-dependent stochastic differential equation (SDE) of form

$$X_s = \xi + \int_t^s b(u, X)du + \int_t^s \sigma(u, X)dW_u$$

with predictable coefficients, including SDEs stopped at boundaries or subject to regime switching, and ask for the Malliavin Besov regularity of their solution. For example, the Besov regularity gives information about the behaviour of the chaos decomposition of the X_s and this behaviour is exploited in simulations of SDEs and BSDEs in the literature. The map $\xi \mapsto X_s$ is not linear, but the interpolation method is, so the investigation of the propagation of the Besov regularity starting from ξ is not obvious. We provide general results using a recent coupling method [1] and potentials, provide upper and lower bounds for the regularity in the case of stopped SDEs and SDEs subject to regime switching. Finally we present a positive result when the coefficient σ is not Lipschitz continuous exploiting a condition on the Ciesielski Schauder expansion of σ, and close with an open problem regarding Hölder continuous coefficients (b, σ).

This is joint work [2] with Xilin Zhou.

Bibliography

[1] S. Geiss and J. Ylinen (2021). Decoupling on the Wiener Space, Related Besov Spaces, and Applications to BSDEs. Memoirs AMS 1335 (2021).

[2] S. Geiss and X. Zhou (2025). Regularity of stochastic differential equations on the Wiener space by coupling. arXiv:2412.10836.