Meyers exponent rules the first-order approach to second-order elliptic boundary value problems

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Since the early 2000s, major progress has been made on boundary value problems for second-order elliptic equations in divergence form with transversally independent—though possibly complex—coefficients in the upper half-space, thanks to the first-order approach. The key idea is to reformulate the equation algebraically as a first-order system, in analogy with how harmonic functions in the plane relate to the Cauchy–Riemann system in complex analysis. The required harmonic analysis stems from techniques developed in the resolution of the Kato conjecture.

When it comes to a priori estimates, solvability, and well-posedness in L^p-topologies, the state of the art is as follows: one gets beautiful results when p lies in an interval around 2, whose upper endpoint $p_+(DB)$ is determined by L^p-extrapolation of the bounded functional calculus for a certain perturbed Dirac operator at the boundary. This is the synthesis of work of Amenta, Auscher, Mourgoglou, and Stahlhut.

Yet the precise nature of this threshold $p_+(DB)$ has remained mysterious. Is it merely an artefact of the first-order method? Or does it capture something intrinsic to the underlying second-order PDE?

In my talk, I will present joint work with Pascal Auscher and Tim Böhnlein that provides a definitive answer: $p_+(DB)$ coincides exactly with the optimal exponent for weak reverse Hölder estimates on the gradient of local solutions to the elliptic equation. The proof hinges on a simple, but apparently overlooked, connection with operator-valued Fourier multipliers in the tangential direction.