Robust interpolation inequalities via Chebyshev-type integral inequalities

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We establish robust log-convex interpolation inequalities within the scale of Gagliardo seminorms. We achieve this by deriving some Chebyshev-type integral inequalities for general non-synchronous functions. Our primary motivation for establishing these robust interpolation inequalities stems from the study of the asymptotic nonlocal-to-local optimal stability of weak solutions to the boundary Dirichlet problem associated with the regional fractional p-Laplacian. More precisely, if $u_s \in W^{s,p}(\Omega)$ weakly satisfies $(-\Delta)^s_{p,\Omega} u_s = f_s$ in Ω and $\gamma_0^s(u_s) = g_s$ on $\partial\Omega$, with $\frac{1}{p} < s \leq 1$ and $\Omega \subset \mathrm{R}^d$ is bounded Lipschitz, then, under appropriate convergence of the data f_s and g_s as $s \to 1^-$, we establish that $\|u_s - u_1\|_{W^{s,p}(\Omega)} \xrightarrow{s\to 1^-} 0$.